

Problem 1. (Inverse Rational Functions)

Let $f : A \rightarrow B$ be the function given by

$$f(x) = \frac{x^2}{x^2 + 1},$$

where $A = [0, \infty)$ and $B = [0, 1)$.

(a) Find $f'(x)$.

(b) Explain why f is bijective (one-to-one and onto).

(c) Let $g : B \rightarrow A$ be the inverse of f . Compute $g'\left(\frac{1}{10}\right)$.

Problem 2. (Inverse Rational Functions)

Let

$$f(x) = \frac{x}{4 - x^2}.$$

Let g be the branch of inverse of f whose domain contains 0.

(a) Find the domain and range of g .

(b) Sketch the graph of g .

(c) Find $g'\left(\frac{1}{3}\right)$.